Math 20550 - Calculus III
Exam 2 Review

New things to know:

14. Partial Derivatives

14.1. Functions of Several Variables.

- How to find the domain and range of a function of two or three variables.
- How to sketch level curves/level surfaces of a function of two/three variables.

14.2. Limits and Continuity.

- How to show that a limit does not exist
- How to justify that a function of two or three variables is continuous (e.g., is a product or composition of continuous functions, or a quotient of continuous functions when the denominator is not zero)
- How to take a limit of a continuous function of several variables

14.3. Partial Derivatives.

- The limit definition of the partial derivative of a function $f(x, y)$ (resp. $f(x, y, z)$) with respect to one of its variables, at a point (a, b) (resp. (a, b, c)), e.g., $f_x(a, b)$.
- How to compute partial derivatives.
- How to compute higher order partial derivatives.
- Clairaut's Theorem

14.5. The Chain Rule.

- The gradient of a function f of several variables.
- A function f of several variables is differentiable if the components of its gradient ∇f are continuous.
- The chain rule:
  (1) If $h(x_1, ..., x_n) = f \circ \mathbf{G}(x_1, ..., x_n)$, where $f = f(y_1, ..., y_m)$ and $\mathbf{G}(x_1, ..., x_n) = \langle y_1(x_1, ..., x_n), ..., y_m(x_1, ..., x_n)\rangle$, then
  $$\frac{\partial h}{\partial x_i}(x_1, ..., x_n) = \frac{\partial}{\partial x_i} f \circ \mathbf{G}(x_1, ..., x_n) = \nabla f(\mathbf{G}(x_1, ..., x_n)) \cdot \frac{\partial \mathbf{G}}{\partial x_i}(x_1, ..., x_n).$$
  (2) Alternatively, using trees of dependence.
- Implicit differentiation
- Related rates problems

14.6. Directional Derivatives and the Gradient Vector.

- How to compute the directional derivative of a function f in the direction of a vector $\mathbf{v}$.
- At a given point, in what direction the directional derivative of a function f is greatest, what its value is in this direction, and what this means in terms of f increasing/decreasing.
- Be able to find the direction of greatest rate of change of a function at a given point.
- Be able to estimate the gradient (direction and magnitude) of a function at various points, given a contour plot of the function.
- How to find the tangent plane to a surface at a point.
- How to find the normal line to a surface at a point.

14.7. **Maximum and Minimum Values.**

- What it means for a point (a, b) to be a local maximum/local minimum of a function $f(x, y)$.
- What a local minimum value/local maximum value of a function $f(x, y)$ is.
- What an absolute maximum/absolute minimim is.
- What a critical point of a function $f(x, y)$ is.
- What a saddle point of a funciton $f(x, y)$ is.
- How to use the Second Derivatives Test to classify critical points of a function $f(x, y)$.
- How to find the extrema (absolute maxima and absolute minima) of a function $f(x, y)$ is a closed and bounded set D.

14.8. **Lagrange Multipliers.**

- How to use the method of Lagrange multipliers for one constraint (for functions of two and three variables).
- Be able to interpret the Lagrange multipliers theorem graphically, i.e., given a contour plot of a function f, and a level curve of a function g (i.e., the graph of $g = k$ for some constant k), be able to use the picture to get information on the extrema of f subject to the constraint $g = k$ (see figure 1 in section 14.8).
- How to use the method of Lagrange multipliers for two contstraints.

15. Multiple Integrals

15.1. **Double Integrals over Rectangles.**

- If $f(x, y) \geq 0$, then the volume V of the solid that lies above the rectangle R and below the surface $z = f(x, y)$ is given by

$$V = \iint_R f(x, y)\, dA.$$

15.2. **Iterated Integrals.**

- How to compute the double integral

$$\iint_R f(x, y)\, dA$$

where R is the rectangle

$$R = [a, b] \times [c, d] = \{(x, y) \mid a \leq x \leq b, c \leq y \leq d\}.$$

- Fubini's theorem

15.3. **Double Integrals over General Regions.**

- How to integrate a function $f(x, y)$ over a region D which is bounded by some collection of functions.
- Be able to read off the region of integration given an interated integral.
- How to find the volume of a solid bounded by a collection of functions.
- How to use a double integral to find the area of a region.
- Properties of double integrals.